$$\mathfrak{S}_n$$

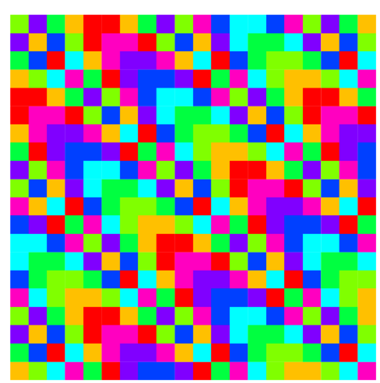
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 3 & 1 & 4 & 2 & 5 & 6 & 7 \end{pmatrix}$$

Mathémagie 2

http://www.discmath.ulg.ac.be/mam/

Michel Rigo

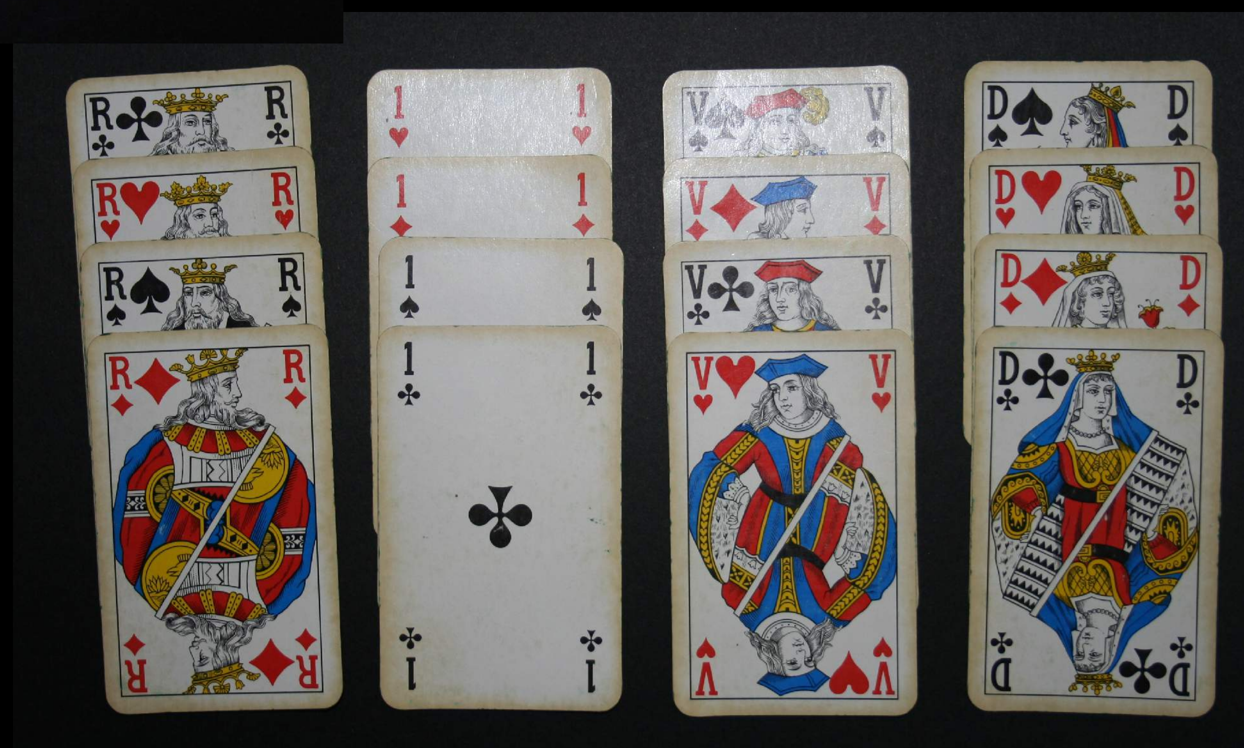
UNIVERSITÉ de Liège



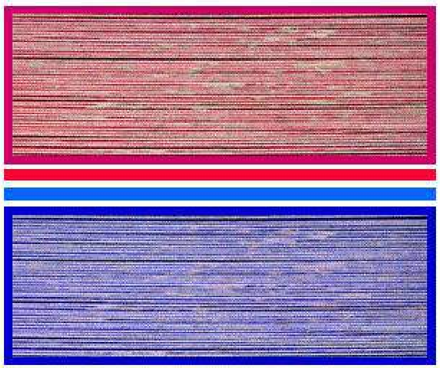
$$\sum_{\sigma \in \mathfrak{S}_n} \mathrm{Nap}(\sigma)$$

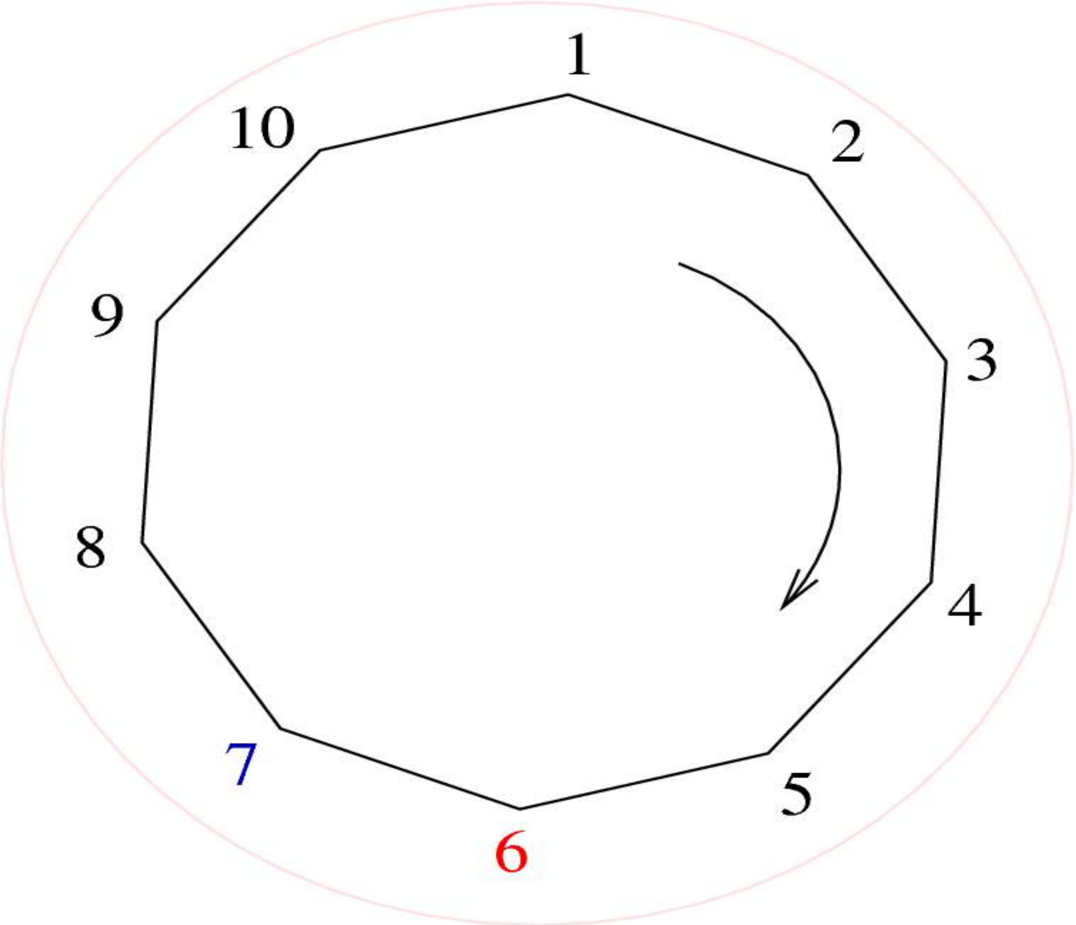
Les appartements royaux …

… une permutation circulaire

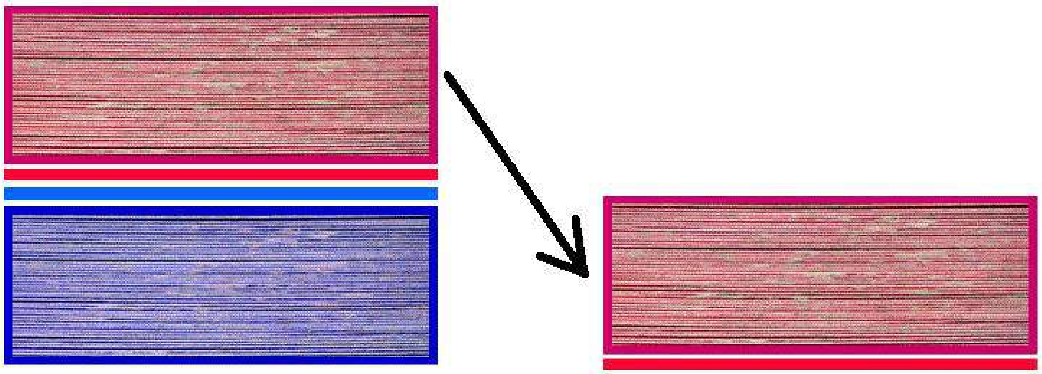


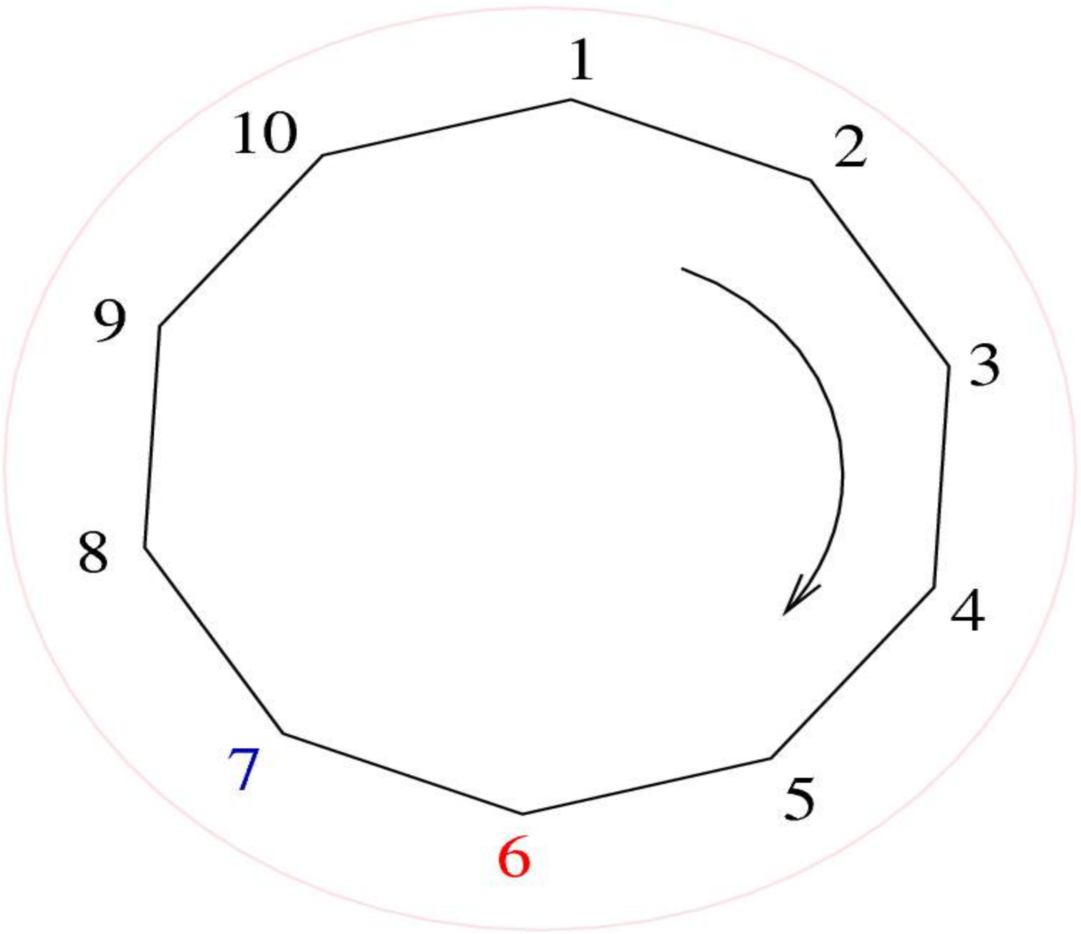
Que se passe-t-il quand on coupe le jeu ?



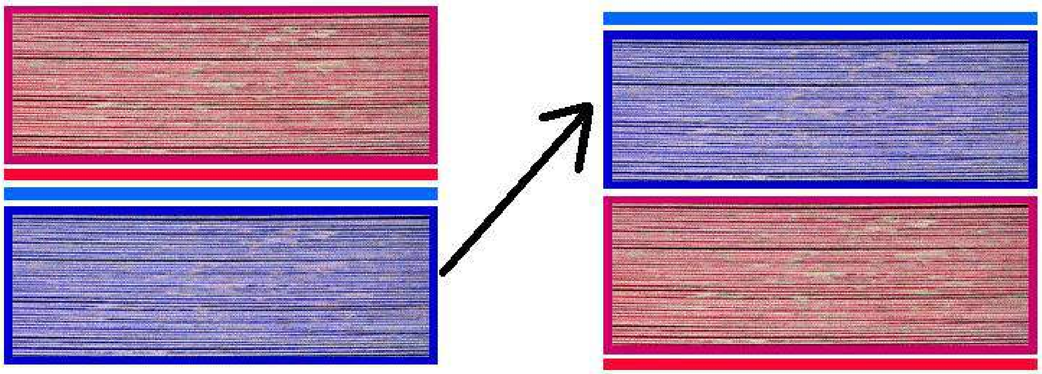


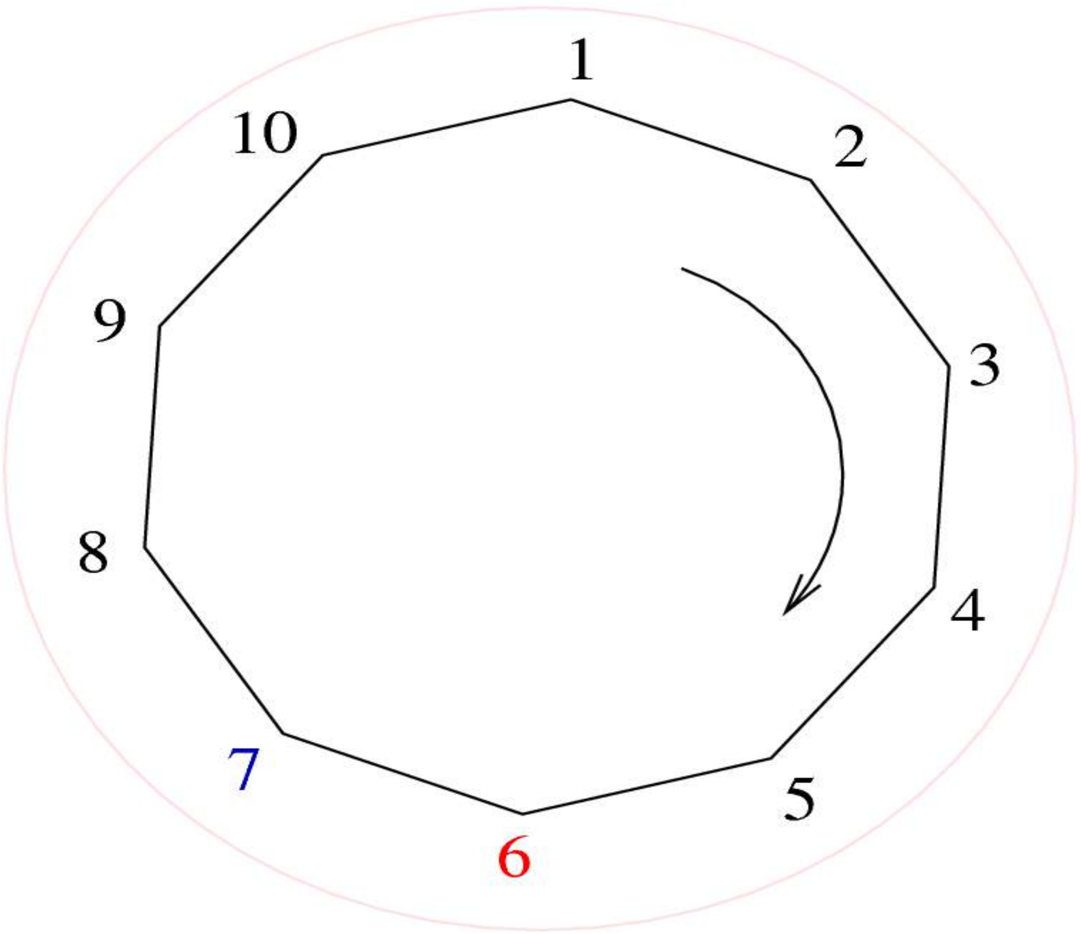
Que se passe-t-il quand on coupe le jeu ?





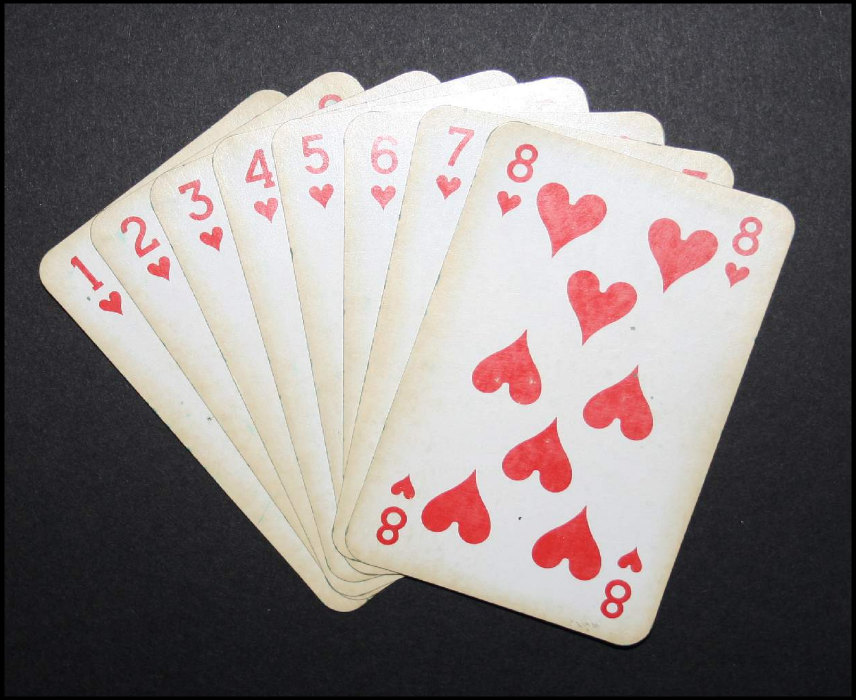
Que se passe-t-il quand on coupe le jeu ?

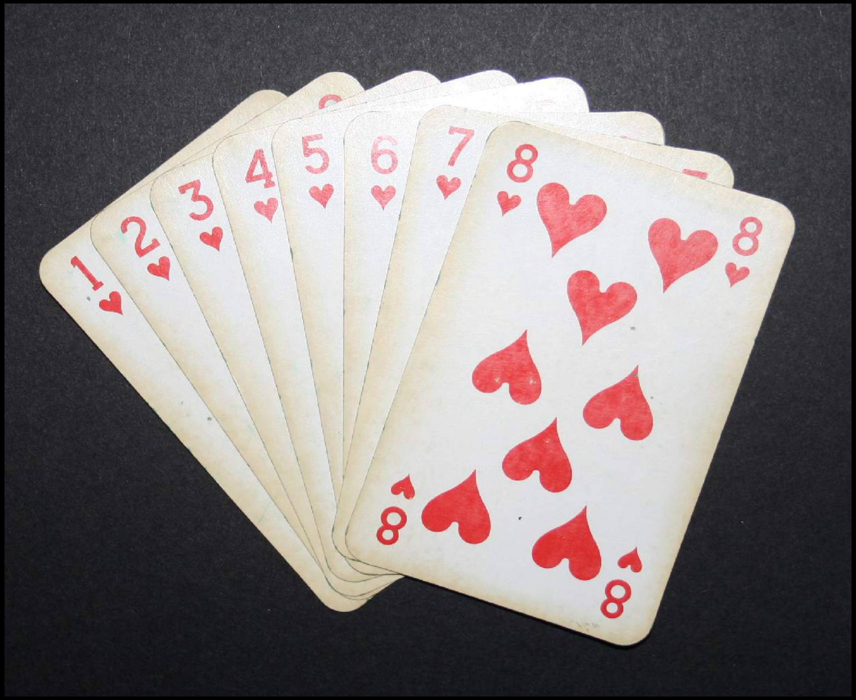


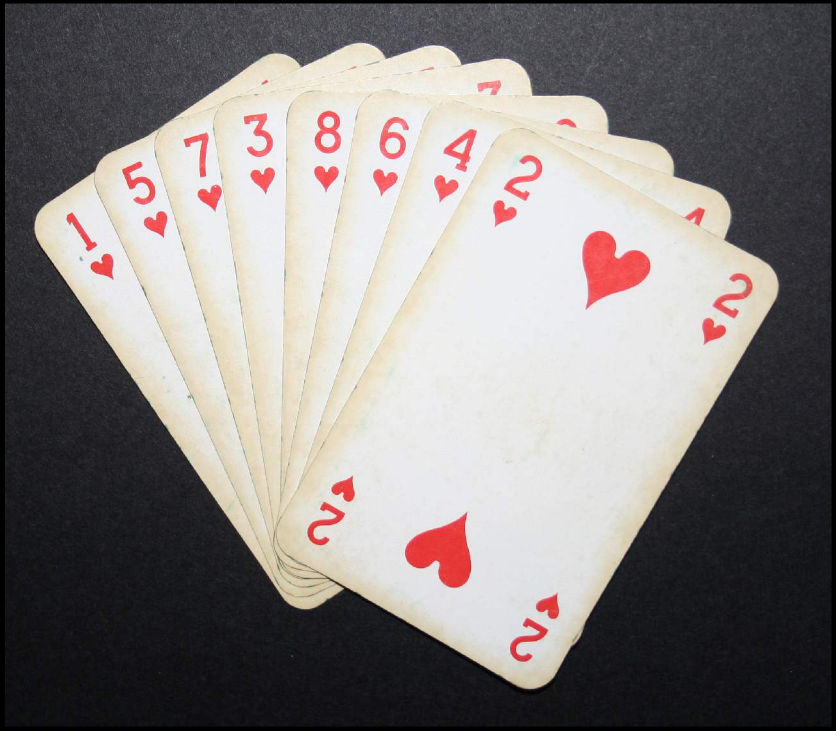


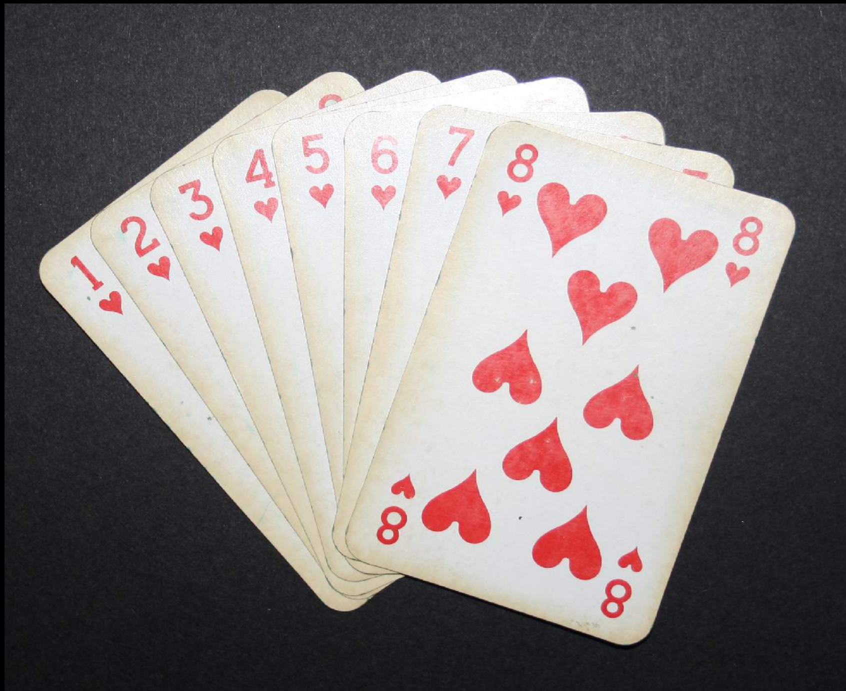
Toute permutation est ...

… un produit de cycles disjoints



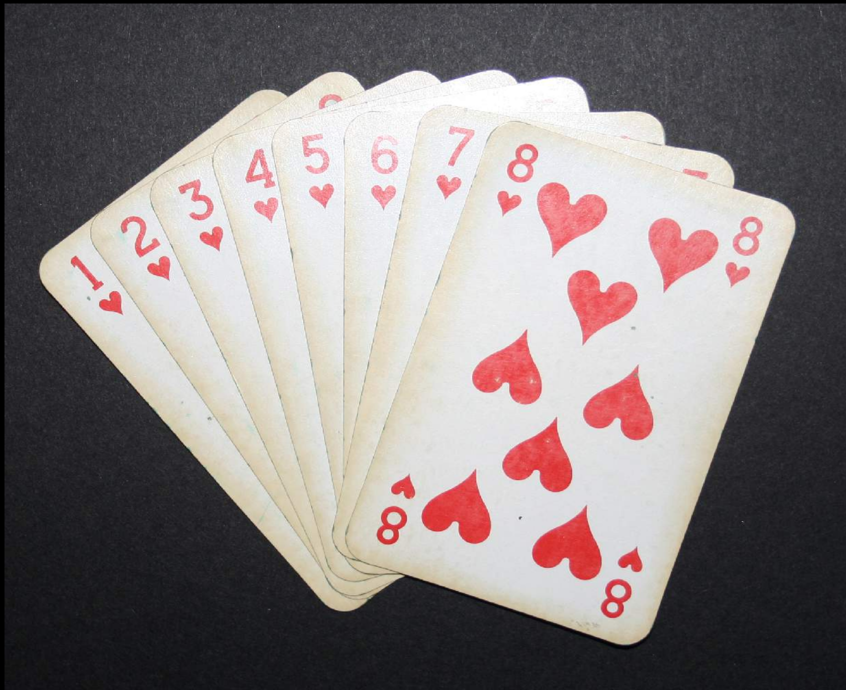






$$\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8\\ 1 & 8 & 4 & 7 & 2 & 6 & 3 & 5\end{pmatrix}$$

$$= \begin{pmatrix}2 & 8 & 5\end{pmatrix}\begin{pmatrix}3 & 4 & 7\end{pmatrix}$$



$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 8 & 4 & 7 & 2 & 6 & 3 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 8 & 5 \end{pmatrix} \begin{pmatrix} 3 & 4 & 7 \end{pmatrix}$$

Exercice : avec 9 cartes

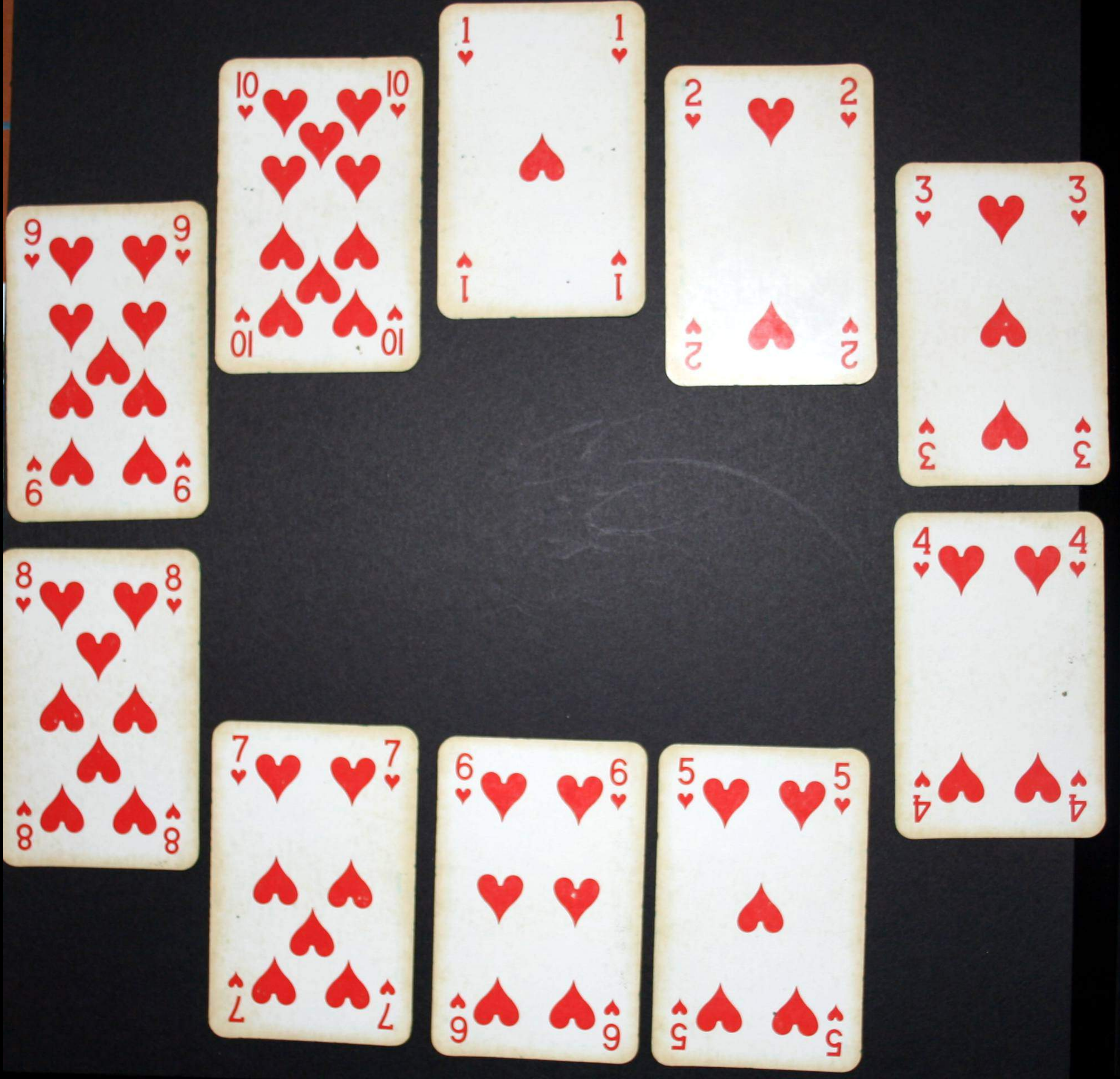
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 9 & 1 & 8 & 4 & 7 & 2 & 6 & 3 \end{pmatrix}$$

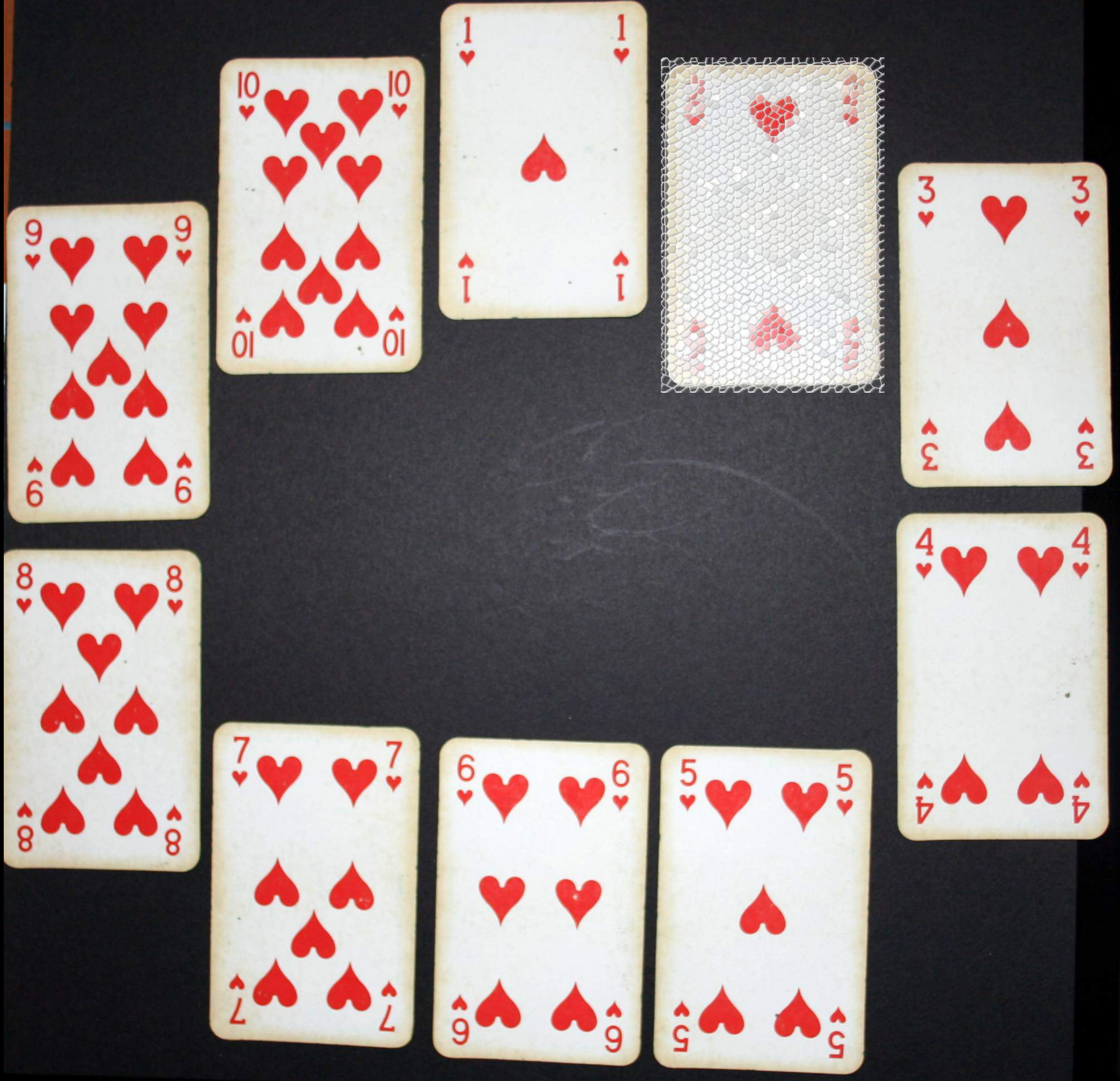
$$= \begin{pmatrix} 1 & 5 & 4 & 8 & 6 & 7 & 2 & 9 & 3 \end{pmatrix}$$

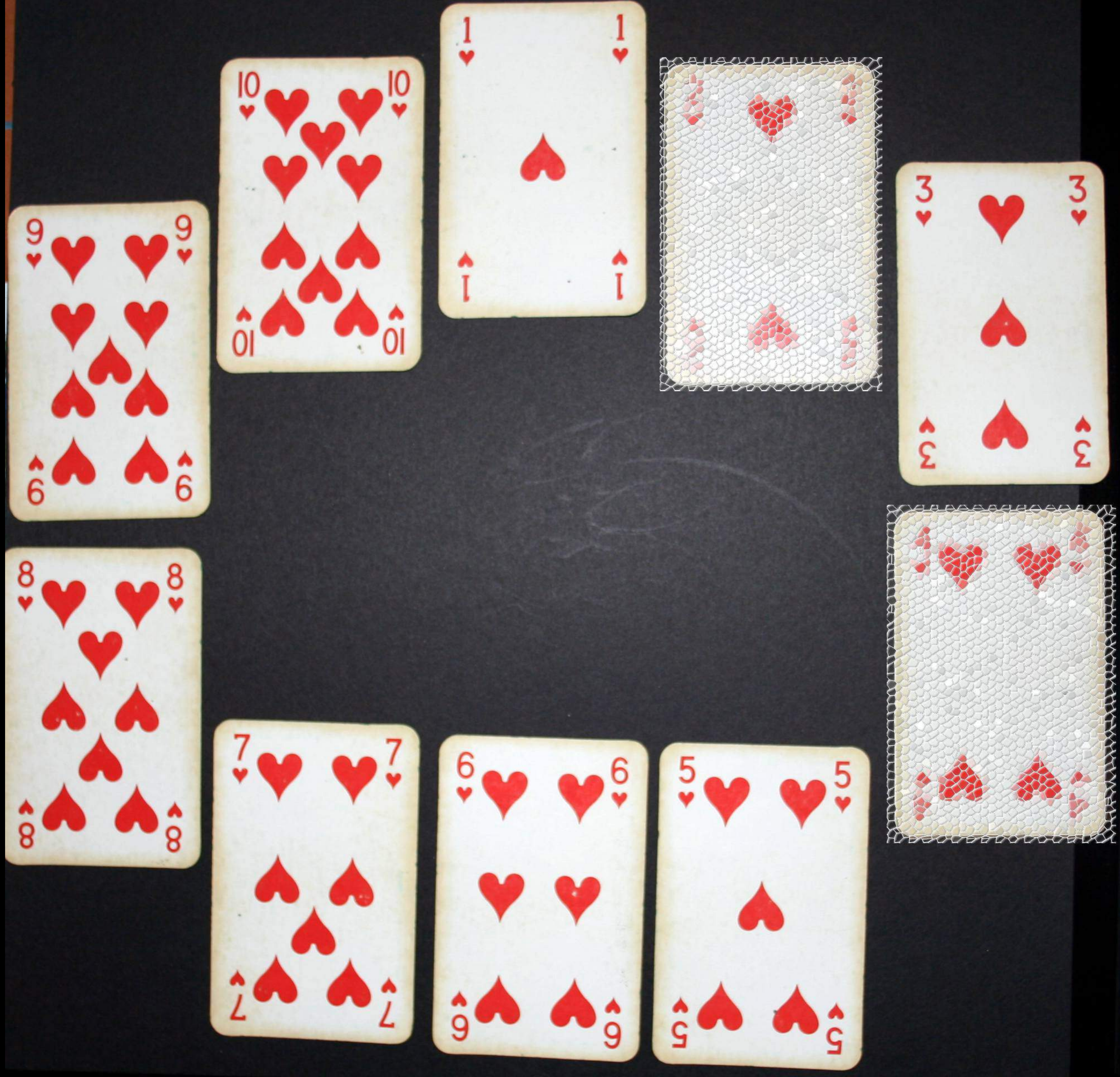
Exercice : avec 10 cartes

$$
\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 10 & 5 & 9 & 1 & 8 & 4 & 7 & 2 & 6 \end{pmatrix}
$$
$$
= \begin{pmatrix} 1 & 3 & 5 \end{pmatrix} \begin{pmatrix} 2 & 10 & 6 & 8 & 7 & 4 & 9 \end{pmatrix}
$$

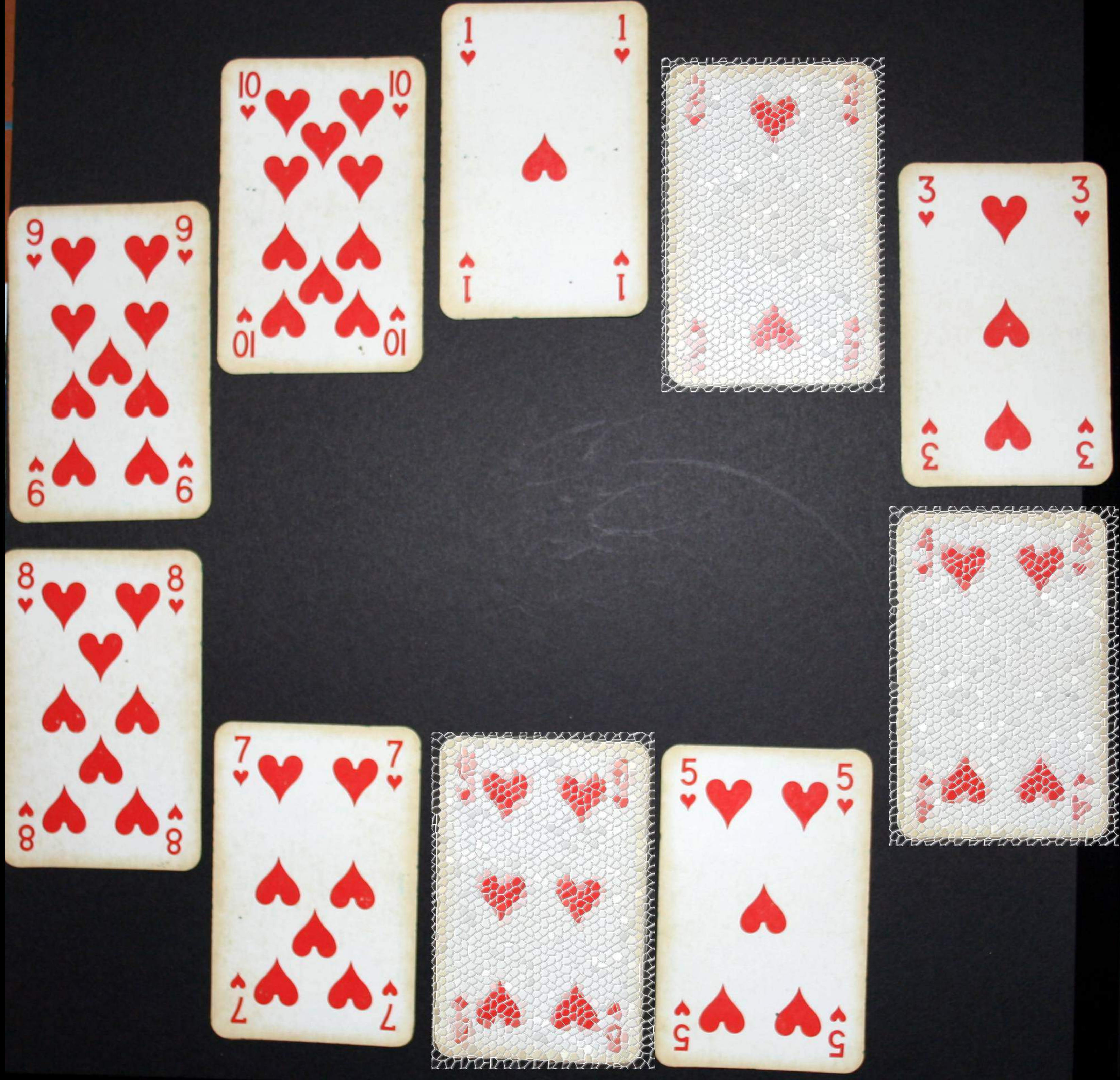
Le problème de Josephus ...

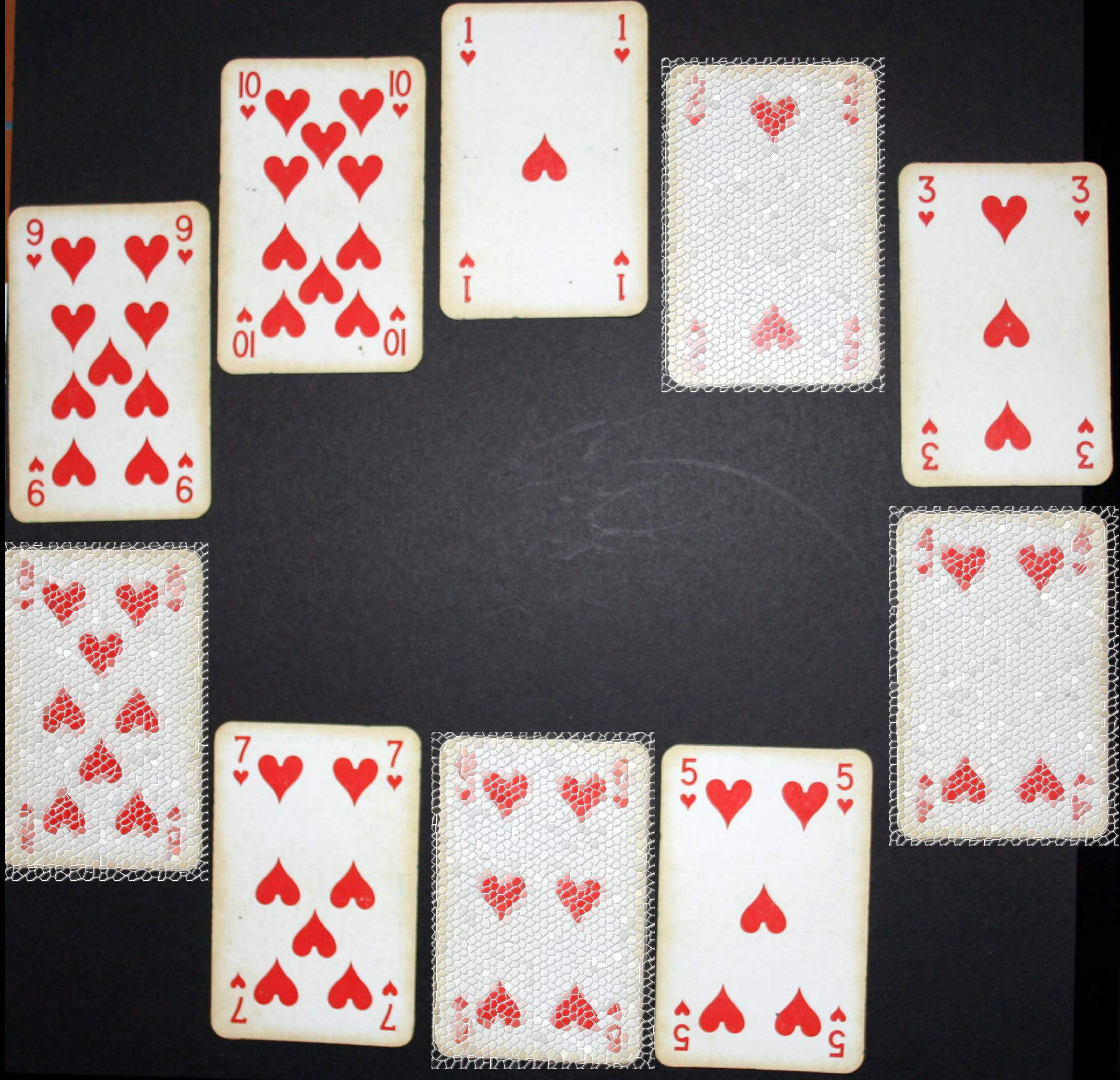


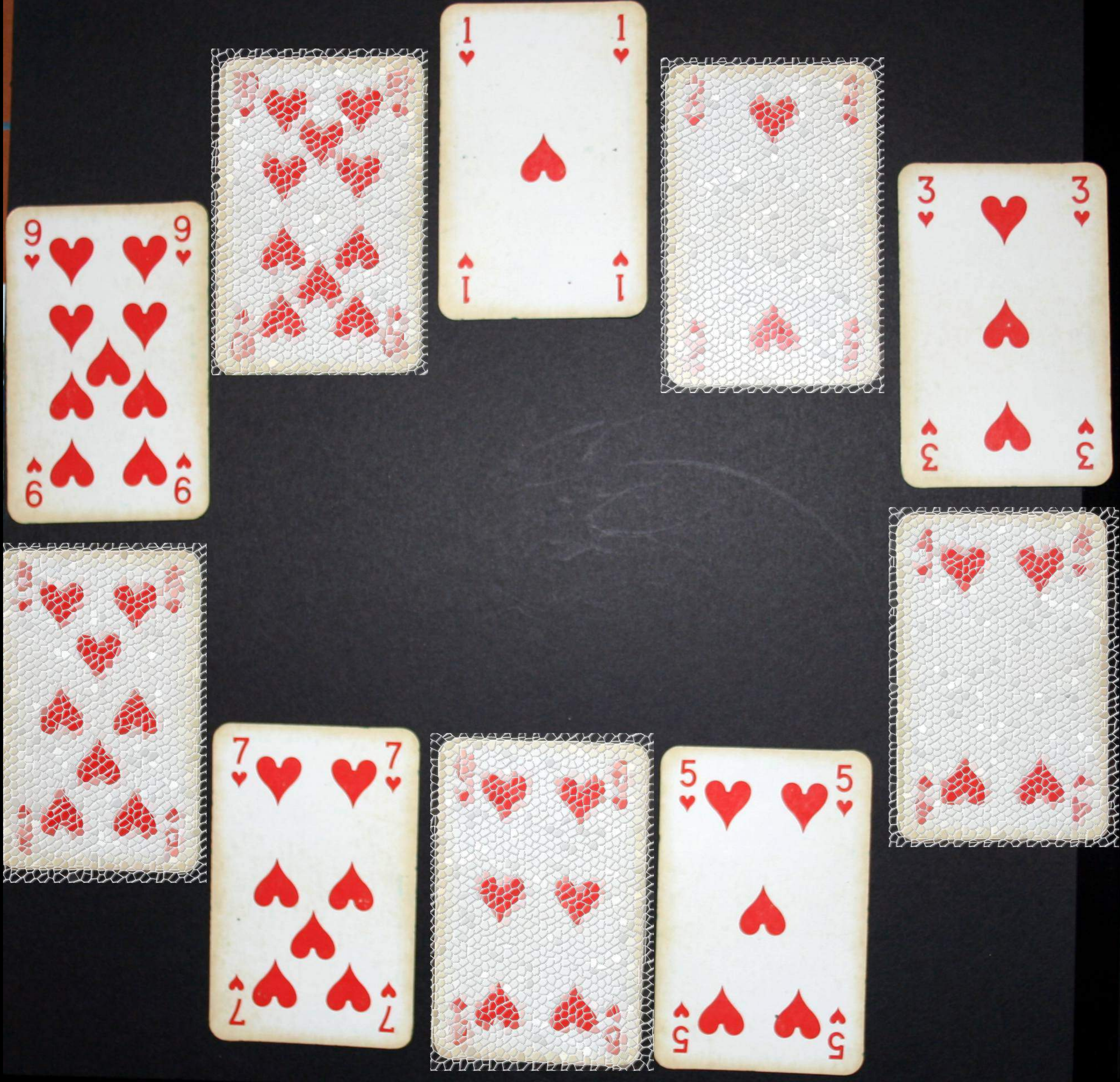


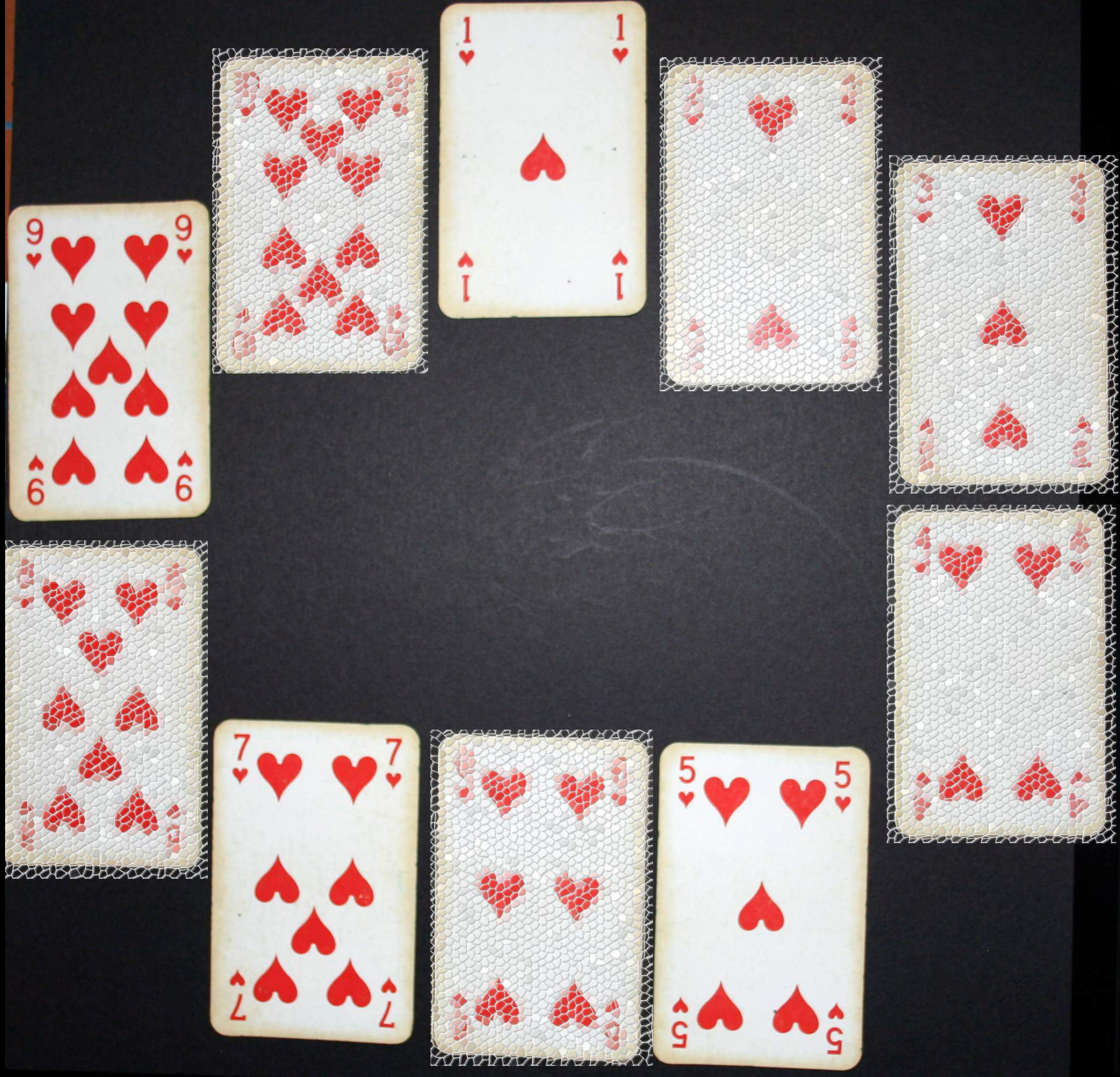


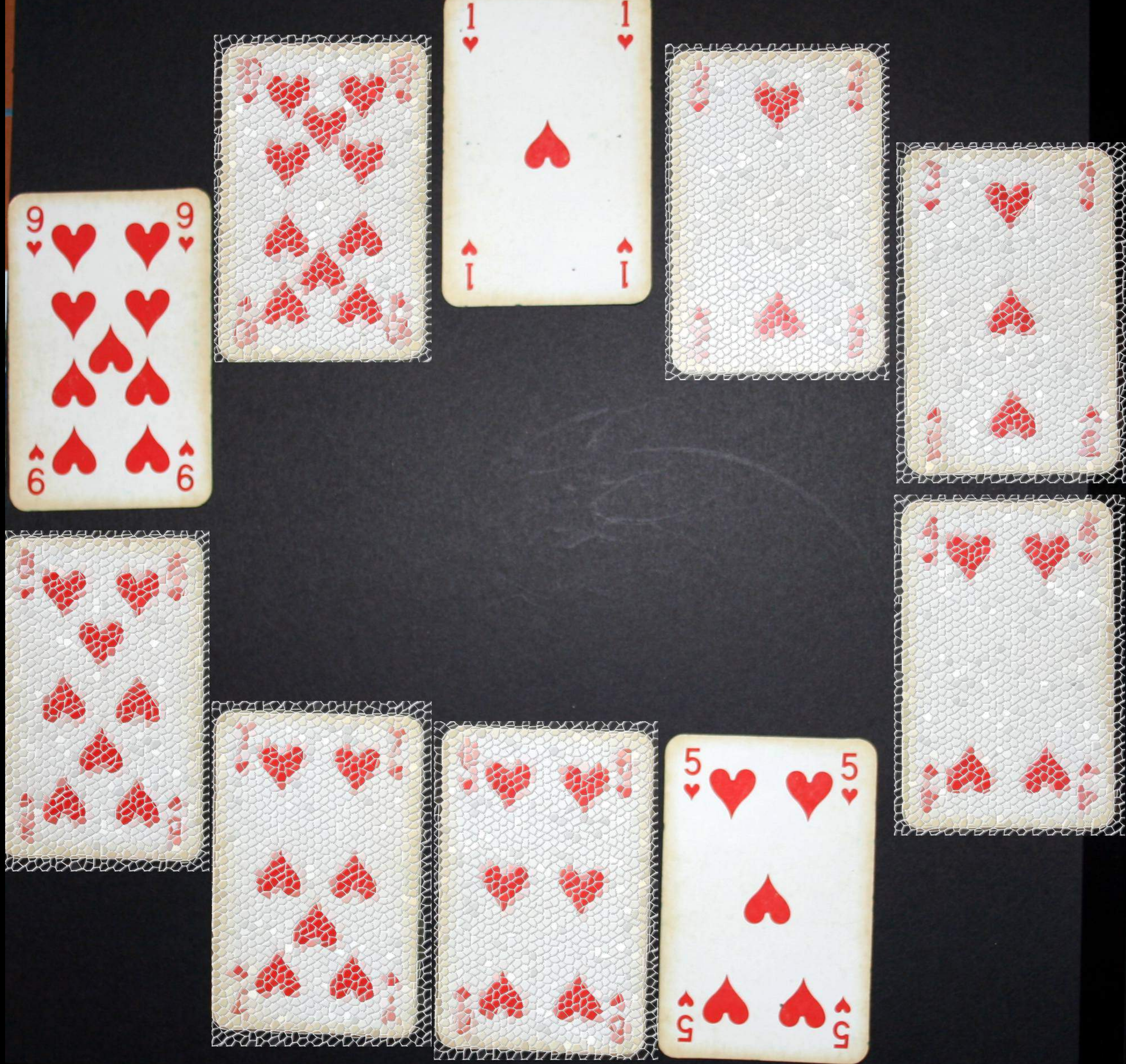
9
10
1
3
8
7
6
5

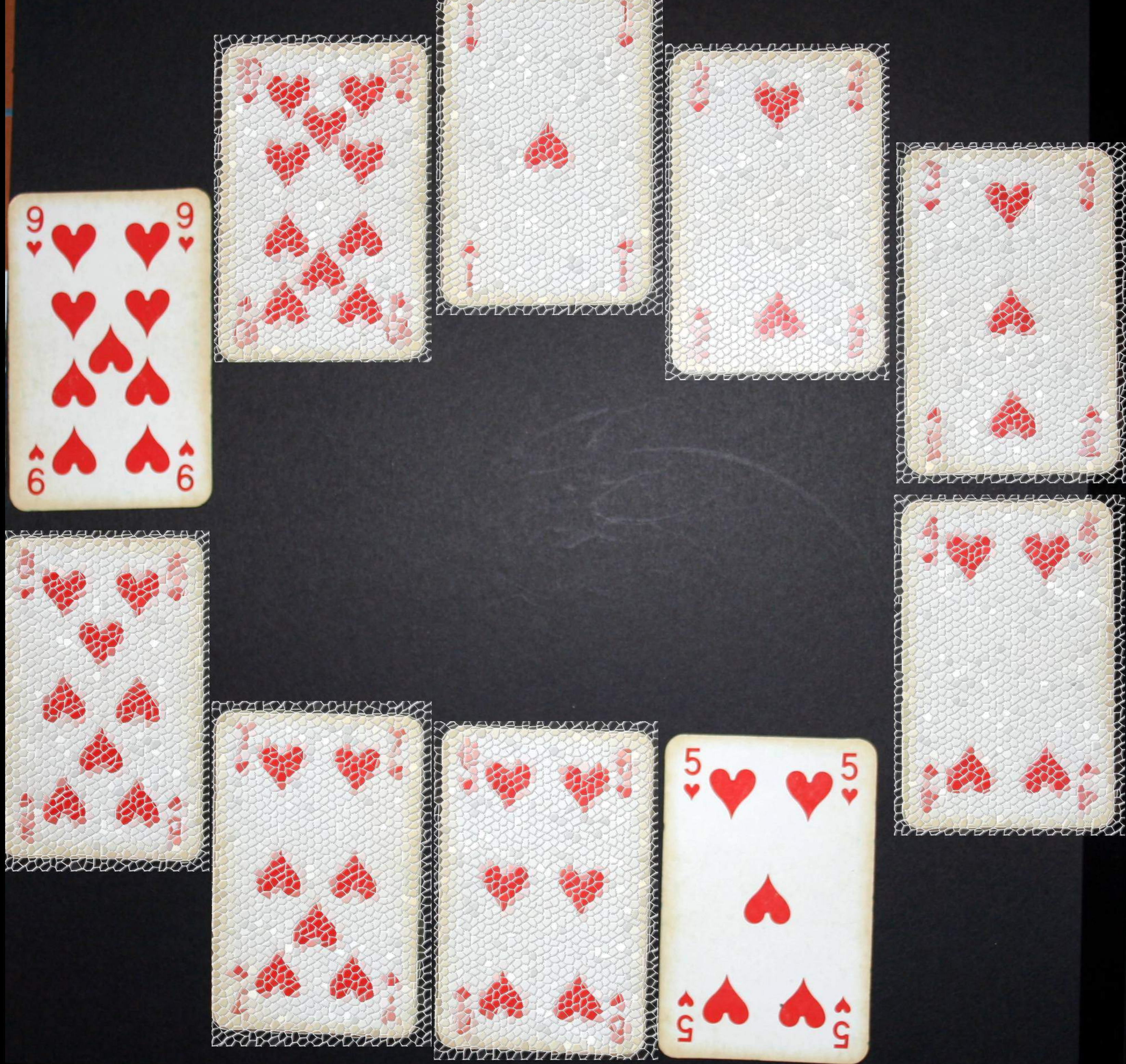

10
1
9
3
8
7
5

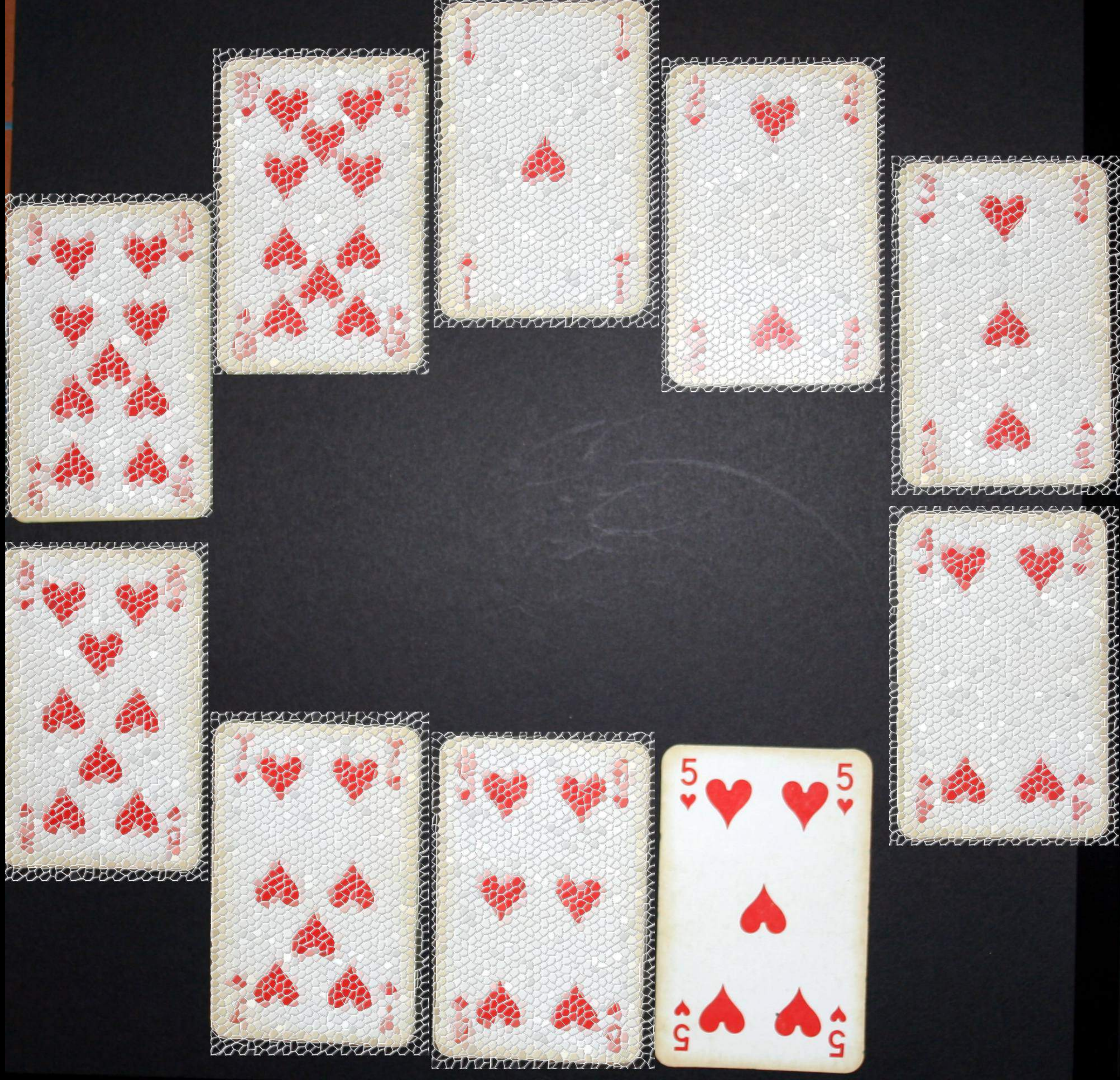









9
5


5

Débutons avec 31 cartes – {1,2,3,4,5} …
et supprimons-en une sur deux …

Quelle sera la dernière carte ?

Josephus

Josephus[n, m]
generates the inverse of the permutation defined by executing every m^{th} member in a circle of n members.

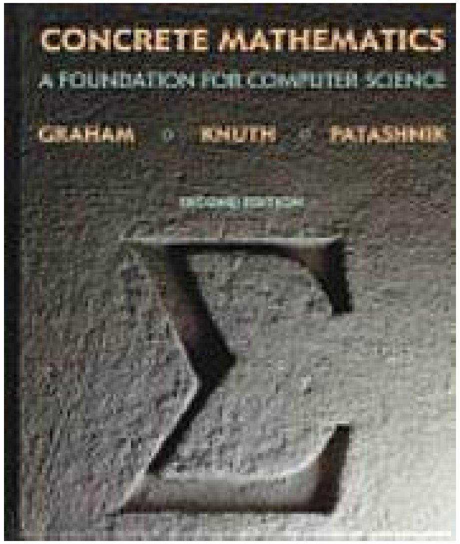
Select a region to grab

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

```
In[1]:=
    Needs["Combinatorica`"]

In[3]:=
    Josephus[10, 2]

Out[3]=
    {8, 1, 6, 2, 10, 3, 7, 4, 9, 5}
```



$$\begin{cases} J(1) = 1 \\ J(2m) = 2\,J(m) - 1, \text{ si } m \geq 1 \\ J(2m+1) = 2\,J(m) + 1, \text{ si } m \geq 1. \end{cases}$$

$$n = 2^m + r \qquad \text{avec } 0 \leq r < 2^m$$

$$\boxed{J(2^m + r) = 2r + 1.}$$

$$\rho_2(n) = x_m x_{m-1} \cdots x_0$$

$$\rho_2(r) = x_{m-1} \cdots x_0$$

$$\rho_2(J(n)) = x_{m-1} \cdots x_0 1$$

$$\rho_2(J(n)) = x_{m-1} \cdots x_0 x_m$$

31	11111	J(31)	11111	31	31-0
30	11110	J(30)	11101	29	30-1
29	11101	J(29)	11011	27	29-2
28	11100	J(28)	11001	25	28-3
27	11011	J(27)	10111	23	27-4
26	11010	J(26)	10101	21	26-5

The Twenty-one cards trick …

… point fixe d'une permutation

1	8	15
2	9	16
3	10	17
4	11	18
5	12	19
6	13	20
7	14	21

1	8	15
2	9	16
3	10	17
4	11	18
5	12	19
6	13	20
7	14	21

MÉLANGE

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21

1	8	15
2	9	16
3	10	17
4	11	18
5	12	19
6	13	20
7	14	21

MÉLANGE

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21

?

1	8	15
2	9	16
3	10	17
4	11	18
5	12	19
6	13	20
7	14	21

MÉLANGE

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21

MÉLANGE

*	*	*
*	*	*
*	2	5
8	11	14
17	20	*
*	*	*
*	*	*

?

1	8	15
2	9	16
3	10	17
4	11	18
5	12	19
6	13	20
7	14	21

MÉLANGE

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21

?

MÉLANGE

*	*	*
*	*	*
*	2	5
8	11	14
17	20	*
*	*	*
*	*	*

?

Le célèbre Leonardo de Pisa ...

$$\begin{cases} F_0 = 1,\ F_1 = 1, \\ F_{n+2} = F_{n+1} + F_n \end{cases}$$

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

3
17
20
37
57
94
151
245
396
641

3
17
20
37
57
94
151
245
396
641

1661

a
b
$a+b$
$a+2b$
$2a+3b$
$3a+5b$
$5a+8b$
$8a+13b$
$13a+21b$
$21a+34b$

$$55\,a + 88\,b$$

a
b
a+b
a+2b
2a+3b
3a+5b
5a+8b
8a+13b
13a+21b
21a+34b

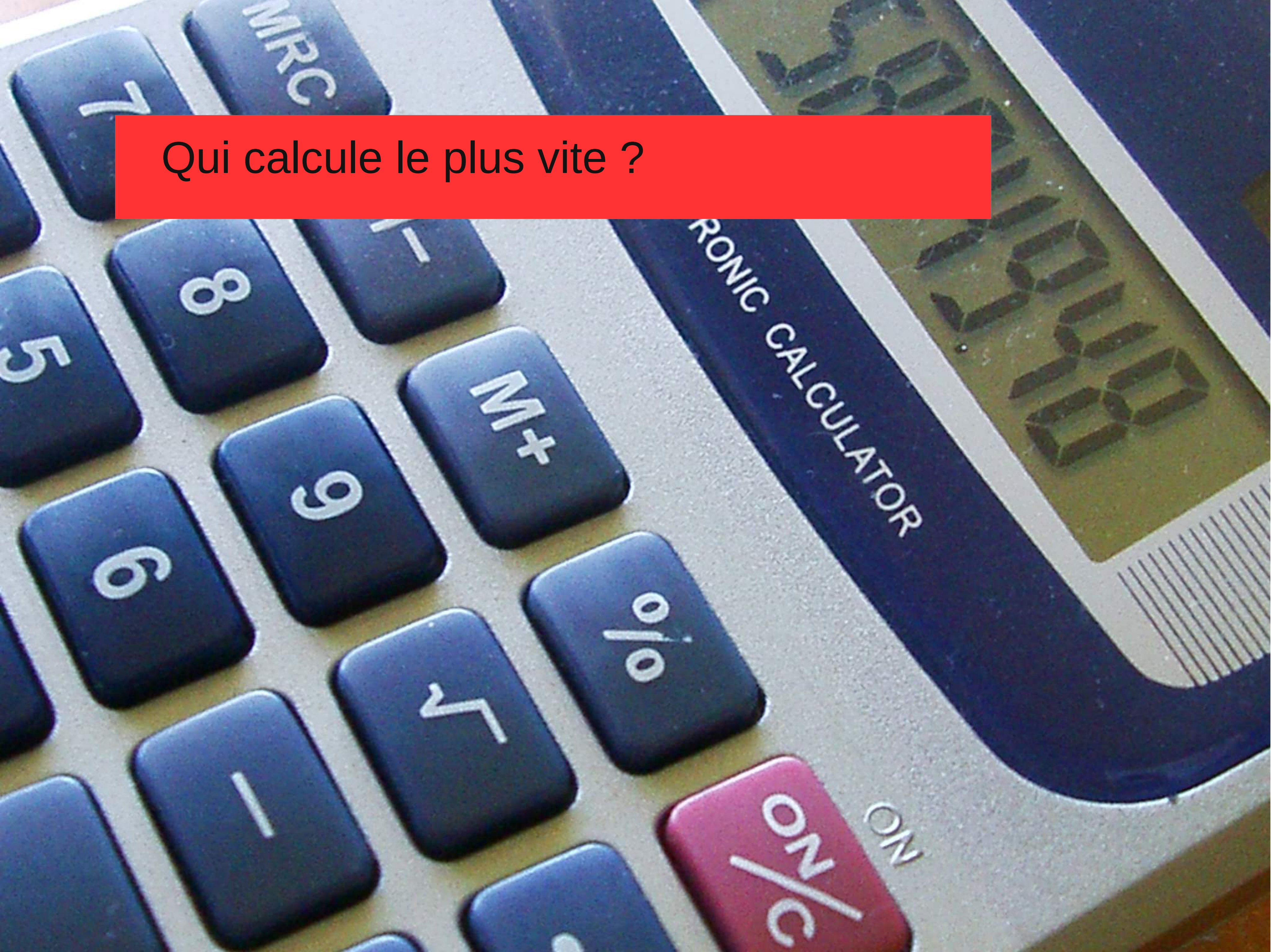
$$\begin{cases} F_0 = 1, \;\; F_1 = 1, \\ F_{n+2} = F_{n+1} + F_n \end{cases}$$

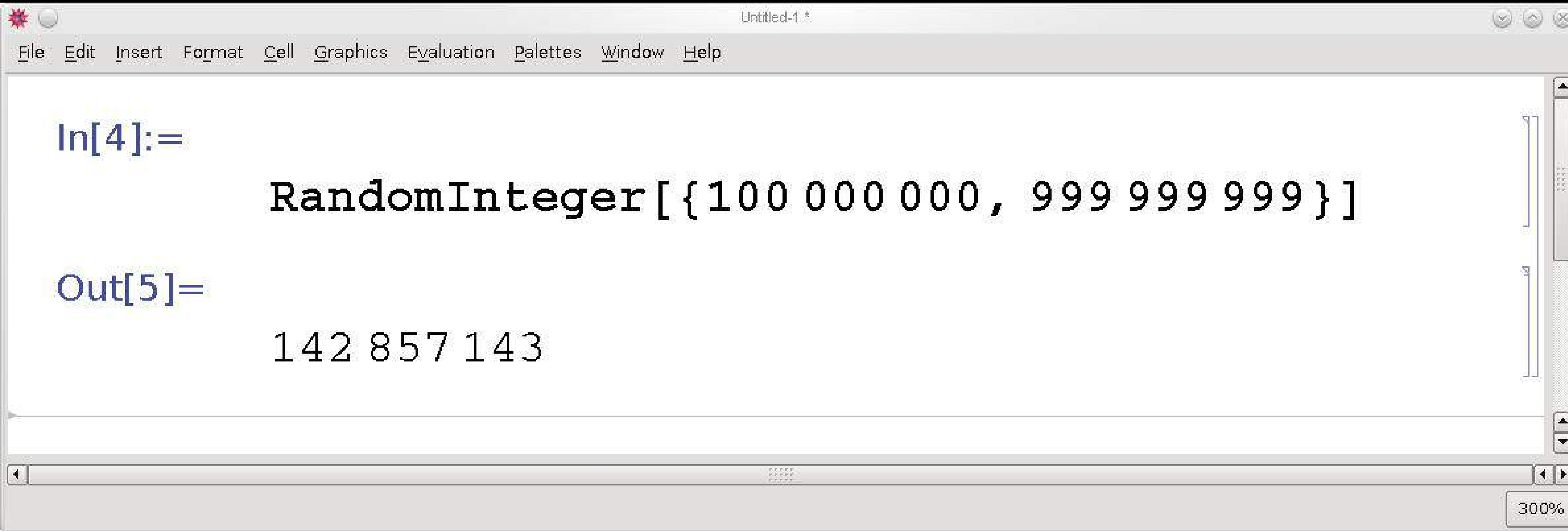
$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

$$\boxed{\sum_{j=0}^{\ell} F_j = F_{\ell+2} - 1}$$

le nombre total de a après ℓ étapes $= F_{\ell-1}$
le nombre total de b après ℓ étapes $= F_\ell - 1$

Qui calcule le plus vite ?





Et le public choisit aussi un nombre de 9 chiffres

1 000 000 001 / 7 = 142 857 143

$$
\begin{array}{lr}
 & 123\ 456\ 789 \\
\times & 1000\ 000\ 001 \\
\hline
 & 123\ 456\ 789\ 123\ 456\ 789
\end{array}
$$